

# KNEE JOINT COORDINATE SYSTEM

V 2.1

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## **1. Revision History**

Revision	Date	Name	Comment
1.0	5-28-14	Colbrunn	Created from JCS12.doc written by Ton van den Bogert
2.0	7-15-14	Colbrunn	Section 4.1 corrected figure to match coordinate system directions.
2.1	7-16-14	Colbrunn	Section 4.1 additionally corrected figure to match coordinate system directions

## **2. Using The Document**

### ***2.1 Purpose***

This document provides the coordinate system transformations used in simVITRO for the knee joint coordinate system (JCS). This will show how to get from digitized data (x, y, z) to transformation matrices and the relative relations of each. All the coordinate systems are linked via the kinematic chain equation and this can be used to provide proper “linkages” between each system. Additional mathematical support can be found at <http://www.euclideanspace.com/>.

simVITRO implements the femur and tibia coordinate system proposed by Pennock & Clark (1990) with one difference: the long axes of femur and tibia may not have the femoral head and ankle joint as reference points. It is important to make these coordinate systems orthogonal and right-handed.

### 3. Rigid Body Definitions

#### 3.1 Tibia

Purpose:

The algorithm will generate the transformation matrix between the Tibia coordinate system TIB and the digitizer world coordinate system WORLD. This matrix is constant as long as the specimen does not move. This same algorithm can be used to create the tibia reference frame relative to a position sensor mounted to the tibia. In that way, the tibia can move in space and the known sensor position relative to the world can be used to calculate the tibia position in space.

Digitized Anatomy: 6 points will be collected

T<sub>1</sub> = Most lateral point on the tibial plateau

T<sub>2</sub> = Most medial point on the tibial plateau

T<sub>3</sub> = Distal tibia point (Medial malleolus of the tibia: most medial point)

T<sub>4</sub> = Distal tibia point (Medial malleolus of the tibia: most medial point)

T<sub>5</sub> = Distal tibia point (Lateral malleolus of the tibia: most lateral point)

T<sub>6</sub> = Distal tibia point (Lateral malleolus of the tibia: most lateral point)

Note that the distal tibial points will be averaged to identify a single distal point for the long axis of the tibia. For this reason they should be spaced equidistant around the available distal anatomy. In addition, the ankle is not always attached to the knee for cadaveric testing and the coordinate system definition needs to be flexible to accommodate the case when the ankle anatomy is available, and when it is not. By digitizing the malleoli twice when the anatomy is available, it produces a similar approximation as digitizing 4 equidistant points around the tibia shaft.

Algorithm:

1. Define Tibia origin at center of tibial plateau.

$$\vec{O}_T = \frac{\vec{T}_1 + \vec{T}_2}{2}$$

2. Define distal point as the average of all the distal points.

$$\vec{T}_{distal} = \frac{\vec{T}_3 + \vec{T}_4 + \vec{T}_5 + \vec{T}_6}{4}$$

3. Define z-axis of the tibia to point superiorly.

$$\vec{T}_z = \frac{\vec{O}_T - \vec{T}_{distal}}{|\vec{O}_T - \vec{T}_{distal}|}$$

4. The temporary x-axis is pointed medially along the tibial plateau and is defined by the normalized vector pointing from T<sub>1</sub> to T<sub>2</sub>.

$$\vec{T}_{x,temp} = \frac{\vec{T}_2 - \vec{T}_1}{|\vec{T}_2 - \vec{T}_1|}$$

5. The y-axis is pointed posteriorly and is defined by the normalized vector orthogonal to T<sub>x,temp</sub> and T<sub>z</sub>.

$$\vec{T}_y = \vec{T}_z \times \vec{T}_{x,temp} \quad \text{Then normalize it.}$$

6. The x-axis is pointed medially and is defined by the normalized vector orthogonal to T<sub>y</sub> and T<sub>z</sub>.

$$\vec{T}_x = \vec{T}_y \times \vec{T}_z \quad \text{Then normalize it.}$$

7. T\_WORLD\_TIB is defined as the rotations and translations from the world coordinate system to the tibia coordinate system. Put the tibia axes and origin in a 4x4 matrix.

$$T_{WORLD,TIB} = \begin{bmatrix} \vec{T}_x & \vec{T}_y & \vec{T}_z & \vec{O}_T \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

### 3.2 Femur

Purpose:

The algorithm will generate the transformation matrix between the Femur coordinate system FEM and the digitizer world coordinate system WORLD. This matrix is constant as long as the specimen does not move. This same algorithm can be used to create the femur reference frame relative to a position sensor mounted to the femur. In that way, the femur can move in space and the known sensor position relative to the world can be used to calculate the femur position in space.

Digitized Anatomy: 6 points will be collected.

- F<sub>1</sub> = Lateral femoral epicondyle
- F<sub>2</sub> = Medial femoral epicondyle
- F<sub>3</sub> = Proximal femur point
- F<sub>4</sub> = Proximal femur point
- F<sub>5</sub> = Proximal femur point
- F<sub>6</sub> = Proximal femur point

Note that the proximal femur points will be averaged to identify a single proximal point for an estimate of the long axis of the femur. For this reason they should be spaced equidistant around the available proximal anatomy. In addition, the femoral head is not always attached to the knee for cadaveric testing and the coordinate system definition needs to be flexible to accommodate the case when the hip anatomy is available, and when it is not. These could be 4 points around the proximal femur shaft, or around the epiphyseal line of the femoral head.

Algorithm:

1. Define femur origin at the mid-epicondylar point.

$$\vec{O}_F = \frac{\vec{F}_1 + \vec{F}_2}{2}$$

2. Define proximal point as the average of all the proximal points.

$$\vec{F}_{proximal} = \frac{\vec{F}_3 + \vec{F}_4 + \vec{F}_5 + \vec{F}_6}{4}$$

3. The x-axis is pointed medially through the epicondyles and is defined by the normalized vector pointing from F<sub>1</sub> to F<sub>2</sub>.

$$\vec{F}_x = \frac{\vec{F}_2 - \vec{F}_1}{|\vec{F}_2 - \vec{F}_1|}$$

4. Define the temporary z-axis of the femur to point superiorly.

$$\vec{F}_{z,temp} = \frac{\vec{F}_{proximal} - \vec{O}_F}{|\vec{F}_{proximal} - \vec{O}_F|}$$

5. The y-axis is pointed posteriorly and is defined by the normalized vector orthogonal to F<sub>x</sub> and F<sub>z,temp</sub>.

$$\vec{F}_y = \vec{F}_{z,temp} \times \vec{F}_x \quad \text{Then normalize it.}$$

6. The z-axis is pointed superiorly and is defined by the normalized vector orthogonal to F<sub>y</sub> and F<sub>x</sub>.

$$\vec{F}_z = \vec{F}_x \times \vec{F}_y \quad \text{Then normalize it.}$$

7. T\_WORLD\_FEM is defined as the rotations and translations from the world coordinate system to the femur coordinate system. Put the femur axes and origin in a 4x4 matrix.

$$T_{WORLD,FEM} = \begin{bmatrix} \vec{F}_x & \vec{F}_y & \vec{F}_z & \vec{O}_F \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

### 3.3 Patella

Purpose:

The algorithm will generate the transformation matrix between the Patella coordinate system PAT and the digitizer world coordinate system WORLD. This matrix is constant as long as the specimen does not move. This same algorithm can be used to create the patella reference frame relative to a position sensor mounted to the patella. In that way, the patella can move in space and the known sensor position relative to the world can be used to calculate the patella position in space.

Digitized Anatomy: 4 points will be collected.

P<sub>1</sub> = Most lateral point

P<sub>2</sub> = Most medial point

P<sub>3</sub> = Most superior point

P<sub>4</sub> = Most inferior point

Algorithm:

1. Define patella origin as the average of the lateral and medial points.

$$\vec{O}_P = \frac{\vec{P}_1 + \vec{P}_2}{2}$$

2. The x-axis is pointed medially and is defined by the normalized vector pointing from the origin to P<sub>2</sub>.

$$\vec{P}_x = \frac{\vec{P}_2 - \vec{O}_P}{\left| \vec{P}_2 - \vec{O}_P \right|}$$

3. Define the temporary z-axis of the patella to point superiorly.

$$\vec{P}_{z,temp} = \frac{\vec{P}_3 - \vec{P}_4}{\left| \vec{P}_3 - \vec{P}_4 \right|}$$

4. The y-axis is pointed posteriorly and is defined by the normalized vector orthogonal to P<sub>x</sub> and P<sub>z,temp</sub>.

$$\vec{P}_y = \vec{P}_{z,temp} \times \vec{P}_x \quad \text{Then normalize it.}$$

5. The z-axis is pointed superiorly and is defined by the normalized vector orthogonal to P<sub>y</sub> and P<sub>x</sub>.

$$\vec{P}_z = \vec{P}_x \times \vec{P}_y \quad \text{Then normalize it.}$$

6. T<sub>WORLD\_PAT</sub> is defined as the rotations and translations from the world coordinate system to the patella coordinate system. Put the patella axes and origin in a 4x4 matrix.

$$T_{WORLD,PAT} = \begin{bmatrix} \vec{P}_x & \vec{P}_y & \vec{P}_z & \vec{O}_P \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## 4. Rigid Body Relative Relationship Definitions

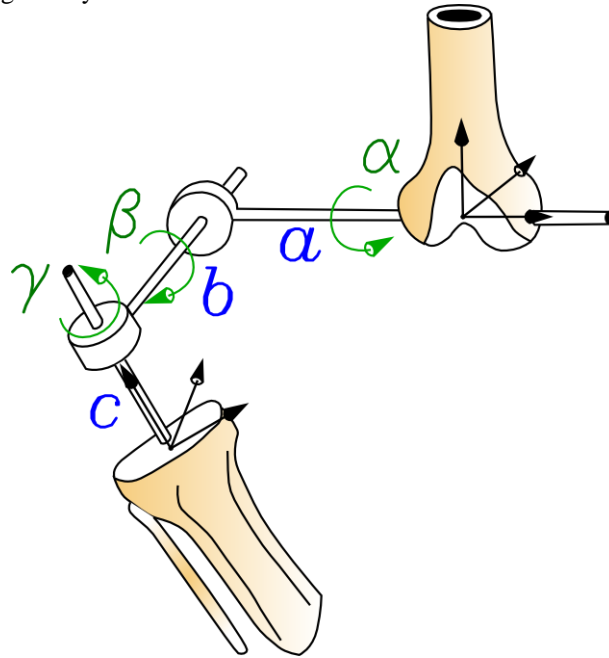
### 4.1 Tibio-Femoral Kinematics

Purpose:

The algorithm will generate the transformation matrix between the femur coordinate system FEM and the tibia coordinate system TIB. This matrix is a function of the 6 DOF JCS rotations and translations.

Method:

simVITRO uses the Joint Coordinate system as defined by Grood and Suntay (1983) and which was also used by Fujie (1996). There are three axes: the flexion axis fixed in the femur, the internal-external rotation axis fixed in the tibia, and the floating axis for varus-valgus rotation which is perpendicular to the other two. Medial translation is measured along the flexion axis, anterior translation along the floating axis and superior-inferior translation measured along the tibia-fixed axis. The position and orientation of these axes are defined in the rigid body definitions.



Convention:

$a$	medial translation of tibia
$b$	posterior translation of tibia
$c$	superior translation of tibia
$\alpha$	flexion
$\beta$	valgus
$\gamma$	internal rotation

Algorithm:

1. Compute

$$\begin{aligned}
\mathbf{T}_{FEM,TIB}(a,b,c,\alpha,\beta,\gamma) &= \begin{pmatrix} 1 & 0 & 0 & a \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & 1 & 0 & b \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 & 0 \\ \sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} \cos\beta & 0 & \sin\beta & a \\ \sin\alpha \sin\beta & \cos\alpha & -\sin\alpha \cos\beta & b \cos\alpha \\ -\cos\alpha \sin\beta & \sin\alpha & \cos\alpha \cos\beta & b \sin\alpha \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 & 0 \\ \sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} \cos\beta \cos\gamma & -\cos\beta \sin\gamma & \sin\beta & c \sin\beta + a \\ \sin\alpha \sin\beta \cos\gamma + \cos\alpha \sin\gamma & -\sin\alpha \sin\beta \sin\gamma + \cos\alpha \cos\gamma & -\sin\alpha \cos\beta & -c \sin\alpha \cos\beta + b \cos\alpha \\ -\cos\alpha \sin\beta \cos\gamma + \sin\alpha \sin\gamma & \cos\alpha \sin\beta \sin\gamma + \sin\alpha \cos\gamma & \cos\alpha \cos\beta & c \cos\alpha \cos\beta + b \sin\alpha \\ 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}$$

## 4.2 Patello-Femoral Kinematics

Purpose:

The algorithm will generate the transformation matrix between the femur coordinate system FEM and the patella coordinate system PAT. This matrix is a function of the 6 DOF JCS rotations and translations.

Method:

simVITRO uses the Joint Coordinate system as defined by Grood and Suntay (1983) for tibio-femoral kinematics and apply them to patella-femoral kinematics. There are three axes: the flexion axis fixed in the femur, the internal-external rotation axis fixed in the patella, and the floating axis for varus-valgus rotation which is perpendicular to the other two. Medial translation is measured along the flexion axis, anterior translation along the floating axis and superior-inferior translation measured along the tibia-fixed axis. The position and orientation of these axes are defined in the rigid body definitions.

Convention:

$a$  medial translation of patella  
 $b$  posterior translation of patella  
 $c$  superior translation of patella  
 $\alpha$  flexion  
 $\beta$  valgus  
 $\gamma$  internal rotation

Algorithm:

1. Compute

$$\begin{aligned}
 \mathbf{T}_{FEM,PAT}(a,b,c,\alpha,\beta,\gamma) &= \begin{pmatrix} 1 & 0 & 0 & a \\ 0 & \cos \alpha & -\sin \alpha & 0 \\ 0 & \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \beta & 0 & \sin \beta & 0 \\ 0 & 1 & 0 & b \\ -\sin \beta & 0 & \cos \beta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \gamma & -\sin \gamma & 0 & 0 \\ \sin \gamma & \cos \gamma & 0 & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} \cos \beta & 0 & \sin \beta & a \\ \sin \alpha \sin \beta & \cos \alpha & -\sin \alpha \cos \beta & b \cos \alpha \\ -\cos \alpha \sin \beta & \sin \alpha & \cos \alpha \cos \beta & b \sin \alpha \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \gamma & -\sin \gamma & 0 & 0 \\ \sin \gamma & \cos \gamma & 0 & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} \cos \beta \cos \gamma & -\cos \beta \sin \gamma & \sin \beta & c \sin \beta + a \\ \sin \alpha \sin \beta \cos \gamma + \cos \alpha \sin \gamma & -\sin \alpha \sin \beta \sin \gamma + \cos \alpha \cos \gamma & -\sin \alpha \cos \beta & -c \sin \alpha \cos \beta + b \cos \alpha \\ -\cos \alpha \sin \beta \cos \gamma + \sin \alpha \sin \gamma & \cos \alpha \sin \beta \sin \gamma + \sin \alpha \cos \gamma & \cos \alpha \cos \beta & c \cos \alpha \cos \beta + b \sin \alpha \\ 0 & 0 & 0 & 1 \end{pmatrix}
 \end{aligned}$$

### 4.3 Knee Joint Kinetics – Tibia Loads

Purpose:

Knee joint kinetics are typically reported in one of two reference frames. simVITRO uses the convention of the loads being reported in the tibia coordinate system rather than the floating joint coordinate system.

Method:

simVITRO uses the tibia coordinate system as defined above for directions and orientations of the joint kinetics. The kinetics are defined as the loads exerted by the femur upon the tibia. Clinical terminology is based on **external** forces applied to the foot (as in a physical exam). This is equal and opposite to the internal loads exerted by the femur. Therefore, the clinical labels for our variables should be as follows:

Convention:

Note that the tibia axes are defined as follows:

X: medial

Y: posterior

Z: superior

$F_x$  Lateral drawer

$F_y$  Anterior drawer

$F_z$  Distraction

$M_x$  External extension moment (will be small unless in hyperextension)

$M_y$  External varus moment

$M_z$  (External) external rotation moment

The above naming convention may be confusing in terms of sign conventions and the definition of the loads. Here is an example to help further clarify the definition.

*Anterior Drawer:* In a clinical exam, with a physician sitting and facing a patient sitting on a table, the physician will pull the tibia toward them. This is an anterior drawer in which the tibia moves anterior relative to the femur (anterior translation). This activity is functionally equivalent to fixing the tibia and having the femur pull posteriorly. The femur pulling posteriorly is a pull in the positive y axis of the tibia. This is why a positive  $F_y$  force is labeled as an anterior drawer even though it may initially seem backwards. All the other degrees of freedom have a similar sign convention phenomena. It is easiest if think of these loading conditions as clinical based loads during an exam.



## Optimized Femur Coordinate System

### Purpose:

The algorithm will generate the static transformation matrix between the digitized femur coordinate system  $FEM_{dig}$  and the optimized femur coordinate system  $FEM_{opt}$ .

### Method:

simVITRO uses the Joint Coordinate system as defined by Grood and Suntay (1983) for tibio-femoral kinematics, yet it is well understood that the reported kinematics can have a large variance due to variance in anatomical landmark digitization. It is desirable to reduce this variation so that reported kinematic data across specimens and across laboratories can be minimally influenced by the operator's choice of what specific location to digitize. The optimized femur coordinate system is a coordinate system with similar origin and orientations as the digitized coordinate system, but refined by a functional loading test. The assumption made is that when the knee is passively flexed, 0-90° with 100 N of compression and all other off axis loads minimized, the kinematics of the joint should have a minimal excursion of translations and rotations (with the exception of the flexion axis.) The knee is not a hinge joint, and these off axis kinematics should not be zero. However, if they are monotonically increasing under the passive loading state then the coordinate system is likely to introduce undesirable variation compared to other specimens. Some robot testing laboratories compensate for this variation in reported kinematics by running a passive loading cycle and then storing these kinematics as the offset for each 1° flexion angle increment. Then, during testing at various flexion angles the offsets are subtracted from the measured kinematics to produce the kinematics that are reported in their manuscripts. While this serves to minimize specimen to specimen variation, it also masks natural knee joint kinematics and effectively forces the knee joint kinematics to be treated as a hinge joint. It also creates 90 different coordinate systems in the knee joint. Masking unique kinematics features of the knee joint is not an ideal choice for minimizing the kinematic variations introduced by the digitization process. The algorithm presented below uses the same passive flexion functional loading, but processes the kinematics in such a way to create a single optimized femur coordinate system.

### Algorithm:

1. Perform passive flexion under force feedback control and record joint kinematics.
2. Extract joint kinematics at 6 flexion angles from 15° to 90° in 15° increments.
3. Use a constrained non-linear optimization algorithm to optimize the 3D translation of the origin. Employ the following objective function and minimize Z. Maintain original orientation.

$$Z = \sum_{i=1}^3 c_i X_i$$

Where:

$X_i$  = Range of values for each translation DOF

$c_i$  = Coefficient for each translation range

4. Use a constrained non-linear optimization algorithm to optimize the 3D orientation of the coordinate system. Employ the following objective function and minimize Z. Use optimized origin.

$$Z = \sum_{i=1}^3 c_i X_i$$

Where:

$X_i$  = Range of values for each rotation DOF

$c_i$  = Coefficient for each rotation range

## 4.4 Optimized Tibia Coordinate System

Purpose:

The algorithm will generate the static transformation matrix between the digitized tibia coordinate system  $TIB_{dig}$  and the optimized tibia coordinate system  $TIB_{opt}$ .

Method:

Just as the femur coordinate system can be optimized through a functional loading test to reduce reported kinematics variations, the tibia coordinate system can also be optimized. The current methodology for this is under development and this section is a placeholder for future algorithm development.

Algorithm:

1. TBD

## 4.5 Left Knee Tibio-Femoral Kinematics

Purpose:

The algorithm will describe the differences between the right knee and left knee coordinate systems.

Method:

simVITRO uses the Joint Coordinate System as defined by Grood and Suntay (1983) for tibio-femoral kinematics. The conventions above produce right handed coordinate systems for right knees. However, they produce left handed coordinate systems for left knees. To make the left knee have a right handed coordinate system would require that sign and naming conventions flip depending on which sided specimen was being tested. This also makes it difficult to compare the results of specimens from different sides. For example, a column of left knee lateral translation data would need to be negated and have the name changed from lateral to medial in order to compare it to right knee kinematics. Essentially, to make the comparison it needs to be mirrored. To use two different conventions would become cumbersome and error prone. The solution employed by simVITRO is to control every specimen in a right handed abstraction reference frame. Simply put, if it is a right knee there is no change: Abstraction Reference Frames = Physical Reference Frames. If the knee is left, everything is mirrored behind the scenes so that it is seamless for the user. Abstraction Reference Frames = Mirrored Physical Reference Frames. The advantage for the simVITRO user is that trajectories can easily be created and data can easily be compared, averaged, and post processed independent of which side the specimen was. In addition, as long as it is right knee, simVITRO data can easily be applied to computational models or physical representations of the specimen. The challenge is that when using simVITRO data, and a left knee, the original mirroring problem remains when trying to directly apply the data to computational models or physical representations of the specimen. In these applications it is necessary to define a left knee coordinate system with modified conventions.

Left Knee Conventions:

Tibia Kinematics:

$\vec{T}_{Lx}$  = lateral

$\vec{T}_{Ly}$  = posterior

$\vec{T}_{Lz}$  = superior

Femur Kinematics:

$\vec{F}_{Lx}$  = lateral

$\vec{F}_{Ly}$  = posterior

$\vec{F}_{Lz}$  = superior

Patella Kinematics:

$\vec{P}_{Lx}$  = lateral

$\vec{P}_{Ly}$  = posterior

$\vec{P}_{Lz}$  = superior

Tibio-Femoral Kinematics:

$a_L$  = lateral translation of tibia

$b_L$  = posterior translation of tibia

$c_L$  = superior translation of tibia

$\alpha_L$  = flexion

$\beta_L$  = varus

$\gamma_L$  = external rotation

#### Patello-Femoral Kinematics:

$a_L$  = lateral translation of patella

$b_L$  = posterior translation of patella

$c_L$  = superior translation of patella

$\alpha_L$  = flexion

$\beta_L$  = varus

$\gamma_L$  = external rotation

#### Tibia Kinetics:

$F_x$  = Medial drawer

$F_y$  = Anterior drawer

$F_z$  = Distraction

$M_x$  = External extension moment (will be small unless in hyperextension)\

$M_y$  = External valgus moment

$M_z$  = (External) Internal rotation moment

#### Algorithm:

Within simVITRO, the software operates as if it is controlling a right knee. When a left knee is mounted, we transform all input and output accordingly so that the internal workings of the software can remain the same. The transformations are mirroring operations. Since simVITRO never assumes any particular alignment of the equipment, it is not important which mirror plane is used. Therefore, simVITRO mirrors with respect to the YZ plane, an arbitrary choice. Practically speaking, this means that when data is read in from physical sensors, the data is mirrored prior to use in transformation matrices. In addition, before the data is used to command actuators to move, it needs mirrored back. Here are the conditional algorithms used for a left knee.

1. 3-DOF position sensor input: negate the x value
2. 6-DOF position sensor input: negate x, pitch, and yaw values
3. 6-DOF actuator output: negate x, pitch, and yaw values
4. 6-DOF load sensor input: negate  $F_x$ ,  $M_y$ , and  $M_z$  values
5. 4x4 transformation matrix: negate elements (0,1), (0,2), (0,3), (1,0), and (2,0)

Where the element is in (row, column) form starting with index 0. This particular algorithm should not be used in addition to negating the sensor and actuator inputs and outputs. This is only used for mirroring a matrix built with non-mirrored sensor data.

The data output of simVITRO will always be in a right handed reference frame and, in the case of a left knee, in a right handed abstraction reference frame. In the case where it is necessary to mirror the left knee data back to the left side convention, the same mirroring algorithms must be employed, and the channel names must be modified to fit the left side convention.

## APPENDIX 1: Notations

$\mathbf{r}_P^A = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$	Position of a point P, expressed in coordinate frame A
$\mathbf{T}_{A,B} = \begin{pmatrix} R_{11} & R_{12} & R_{13} & t_x \\ R_{21} & R_{22} & R_{23} & t_y \\ R_{31} & R_{32} & R_{33} & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$	Matrix to transform B-coordinates into A-coordinates: $\mathbf{r}_P^A = \mathbf{T}_{A,B} \cdot \mathbf{r}_P^B$ and $\mathbf{T}_{A,C} = \mathbf{T}_{A,B} \cdot \mathbf{T}_{B,C}$
$\mathbf{R}_{A,B} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix}$	Rotational part of $\mathbf{T}_{A,B}$
$\mathbf{t}_{A,B} = \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$	Translational part of $\mathbf{T}_{A,B}$ . Physical meaning: the position of B's origin expressed in the A reference frame.
$\mathbf{F}^A = \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix}$	Force expressed in components relative to coordinate frame A. The symbol $\mathbf{M}^A$ is defined similarly.

Reference frames:

TIB	Tibia reference frame
FEM	Femur reference frame
PAT	Patella reference frame
WORLD	World coordinate frame. Is attached to the base of the digitizing sensor, in no particular way.

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